Efficient External-Memory Bisimulation on DAGs

Jelle Hellings[1], George H. L. Fletcher[2], and Herman Haverkort[2]
[1] Hasselt University and transnational University of Limburg
[2] Eindhoven University of Technology

May 25, 2012

Introduction

Definitions

Algorithm

Experimental verification

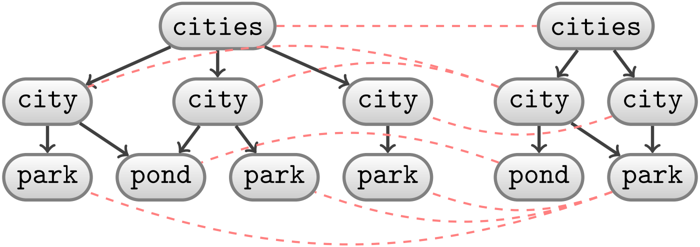
Conclusion

Graphs

- Used as formal model
  - Graph and XML databases
  - Model checking
- Graphs can become large for modern applications
  - Large XML databases
  - Social networks
  - Biological interaction networks

Bisimilarity

- Idea: ‘compress’ graph to speed up computations
- Intuitively: group nodes that ‘reflect the same behavior’



Internal memory algorithm

Fast general algorithm by Paige and Tarjan [1987]

- Runtime complexity: $O(|E| \log(|N|))$
- Memory usage: $O(|N| + |E|)$

Even faster algorithm for DAGs by Dovier et al. [2004]

- Runtime complexity: $O(|N| + |E|)$
- Depends on node ranks

External memory

Cost for reading or writing data from hard disk drive:

Seek move to the correct position on the hard disk drive
Slow: 15ms

Transfer from the correct position read or write the data
'Fast', small blocks in less than 1μs

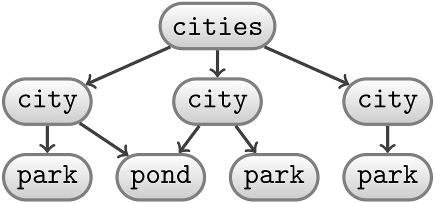
Goal: minimize seeks by transferring large blocks

Node-labeled graphs

Definition

A (node labeled) graph is represented by a triple $G = \langle N, E, l \rangle$:

1. N is a set of nodes,
2. $E \subseteq N \times N$ is a directed edge relation,
3. l is a node-label function



Note

We work with directed acyclic graphs

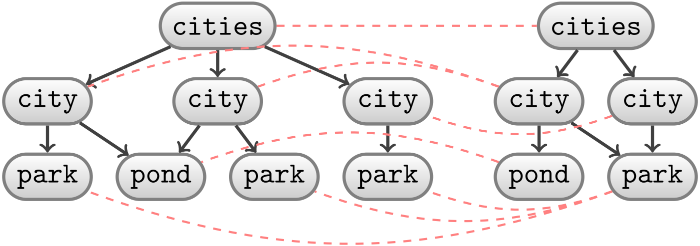
Node bisimilarity

Definition

Nodes n_1 and n_2 are *bisimilar* to each other, denoted $n_1 \approx n_2$, if and only if:

1. the nodes have the same label: $l_1(n_1) = l_2(n_2)$;
2. for every node $n_1' \in \mathit{children}(n_1)$ there is a node $n_2' \in \mathit{children}(n_2)$ such that $n_1' \approx n_2'$; and,
3. for every node $n_2' \in \mathit{children}(n_2)$ there is a node $n_1' \in \mathit{children}(n_1)$ such that $n_1' \approx n_2'$.

Node bisimilarity: example



Node rank

- The rank of a node is the maximum distance to a leaf node
- Rank is denoted by $rank(n)$
  - Rank of leaf nodes is 0
  - Rank of any other node n is $1 + \max_{m \in children(n)} rank(n)$

Fact

$m \approx n$ implies $rank(m) = rank(n)$

Main concepts

- Bisimilarity class is identified by unique identifier
  Bisimilarity identifier
- Set of bisimilarity identifiers of children
  Bisimilarity family
- Label, rank and bisimilarity family of a node
  Bisimilarity decision value

Theorem

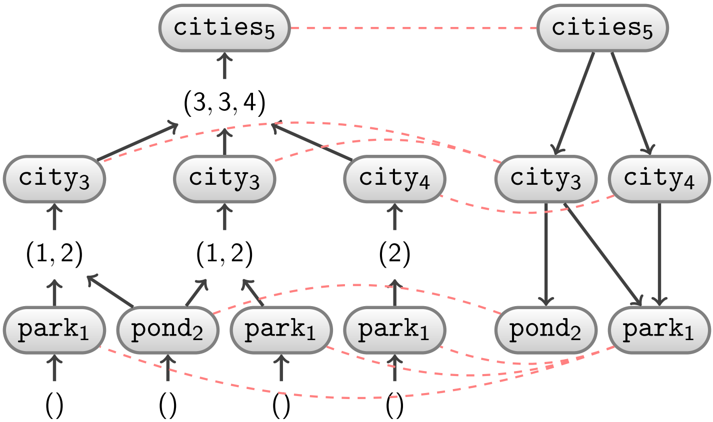
Nodes have equivalent bisimilarity decision value if and only if they are bisimilar equivalent

Algorithmic outline

Require: Directed acyclic graph $G = \langle N, E, l \rangle$.
Ensure: Bisimulation partitions of N.

Sort nodes on rank
for rank $r = 0$ to maximum rank **do**
    Determine bisimilarity decision value of nodes with rank r
    Sort on bisimilarity decision value
    Assign unique bisimilarity identifier to each group
    Send assigned bisimilarity identifiers to parent nodes
end for

Algorithm: example



Practical details

- Use external memory string sorting for grouping
- Use time-forward processing

Theorem

We can compute the bisimilarity equivalence classes of a DAG in $O(\textsc{Sort}(|N| + |E|))$ IOs

Specializations for XML indexing

- We can construct the 1-index in $O(\textsc{Sort}(|N|))$ IOs
- We can construct the $A(k)$-index in $O(\textsc{Sort}(k|N|))$ IOs

Implementation

- C^{++} implementation for DAGs
- C^{++} implementation for 1-index on XML documents
- On top of the STXXL library
  - Provides priority queue and sorting
  - No string sorting; sorting on hash values instead

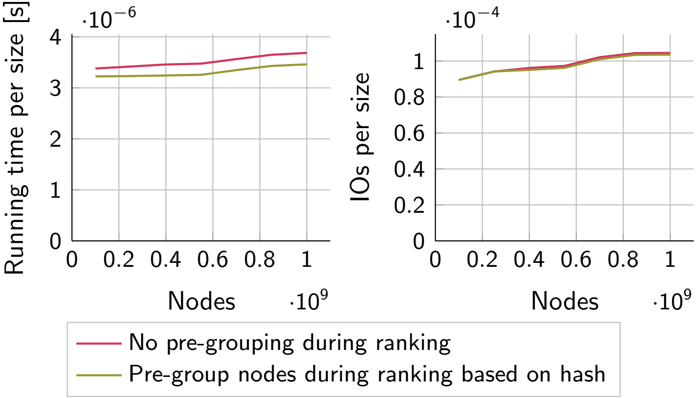
Optimization

During rank partitioning we can already partition better

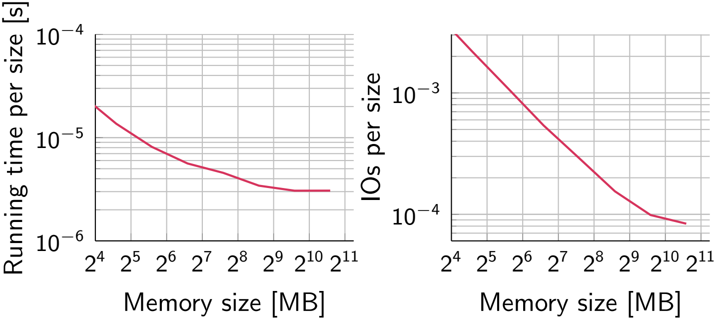
- Include labels
- Partition on ‘structural summaries’

Result: during bisimulation partitioning each group of investigated nodes is smaller.

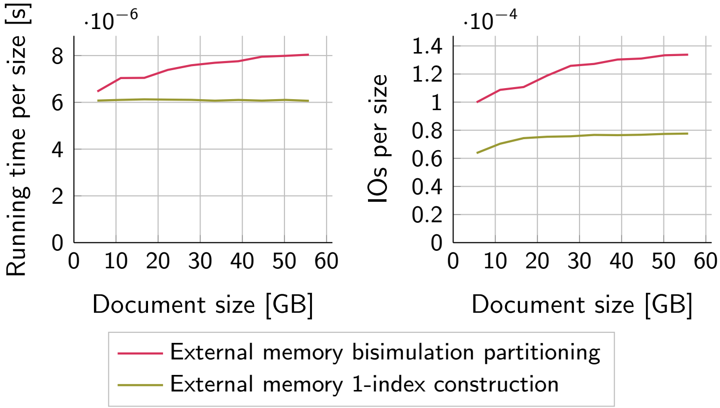
Bisimulation partitioning of DAGs



Performance with respect to internal memory



XML Specialization: the 1-Index



Results

- IO efficient bisimulation partitioning on DAGs
- Specializations for practical XML indices
- Open source implementations in C^{++}
- Empirical validation

Future work

- Generalizing bisimulation partitioning
- Partition maintenance
- Practical output formatting

http://jhellings.nl