Efficient external-memory bisimulation on DAGs

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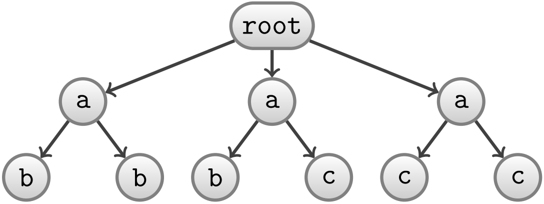
Example: Querying XML data

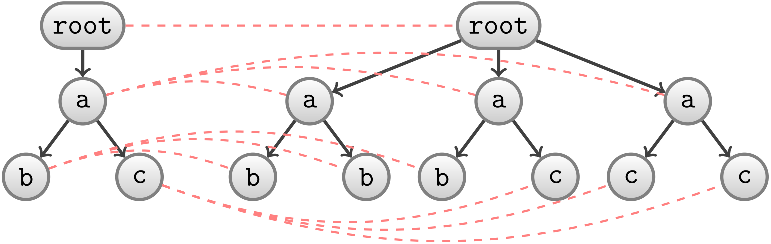
```
<root>
   <a>
      <b/> <b/>
   </a>
   <a>
      <b/> <c/>
   </a>
   <a>
      <c/> <c/>
   </a>
</root>
```



Query 1 does a path `root/a/b` exist?

Query 2 give all nodes reachable by path `root/a/b`.

Graph index



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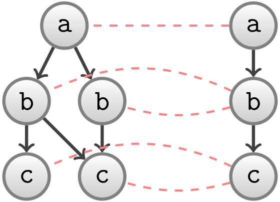
Bisimulation

Definition

Node n bisimulates node m; denoted as $n \approx m$; if and only if:

1. The nodes have the same label,
2. Every child n' of n is bisimulated by a child m' of m, and
3. Every child m' of m is bisimulated by a child n' of n.



Bisimulation partitioning

- Already very well studied
- Fast general algorithm by Paige and Tarjan
    - Runtime complexity: $O(|E|\log(|N|))$
    - Memory usage: $O(|N| + |E|)$

Internal vs External

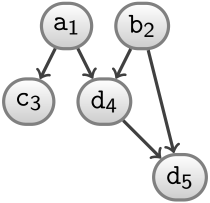
Cost for reading or writing data from hard disk drive:

Seek move to the correct position on the hard disk drive
Slow: 15ms

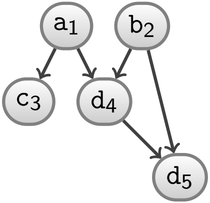
Transfer from the correct position read or write the data
'Fast', small blocks in less than 1μs

Goal: minimize seeks by transferring large blocks

Challenge: graphs



Challenge: graphs



Paige and Tarjan: many edge traversals

Input layout

- Nodes have rank
  Nodes are topologically ordered
- Nodes have identifier
- Nodes has list of identifiers of parents
- Nodes are ordered on identifier and rank

Algorithm - sketch

Input: Directed acyclic graph $G = \langle N, E, l \rangle$.
Output: Bisimulation partition of N.

1: **for all** increasing rank r; starting at $r = 0$ **do**
2: Determine bisimilarity decision value of nodes with rank r
3: Group on equivalent bisimilarity decision values
4: Assign bisimilarity identifiers to each group
5: Send bisimilarity identifiers to parent nodes

Bisimilarity decision value

- Every equivalence class: bisimilarity identifier
- Nodes are bisimilar equivalent iff they have
  - The same label
  - Same set of bisimilarity identifiers of children

Algorithm - sketch

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```
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    Determine bisimilarity decision value of nodes with rank r
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    Send bisimilarity identifiers to parent nodes
```

Grouping on bisimilarity decision values

Assume: set of bisimilarity identifiers of children are ordered

- Bisimilarity decision values are 'strings'
- External memory string sorting

Algorithm - sketch

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Send bisimilarity identifiers to parent nodes

Time-forward processing:

- External memory priority queue
- Ordered on (node identifier, bisimilarity identifier)
- Process nodes in order of their node identifier

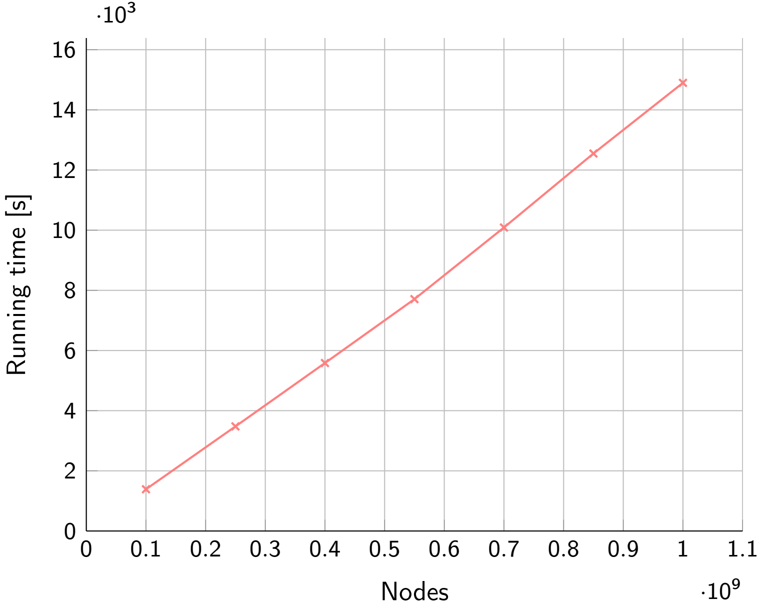
Bisimulation partitioning

- General purpose algorithm
- Topological ordered directed acyclic graphics
- Worst case complexity $O(\mathrm{Sort}(|N| + |E|))$

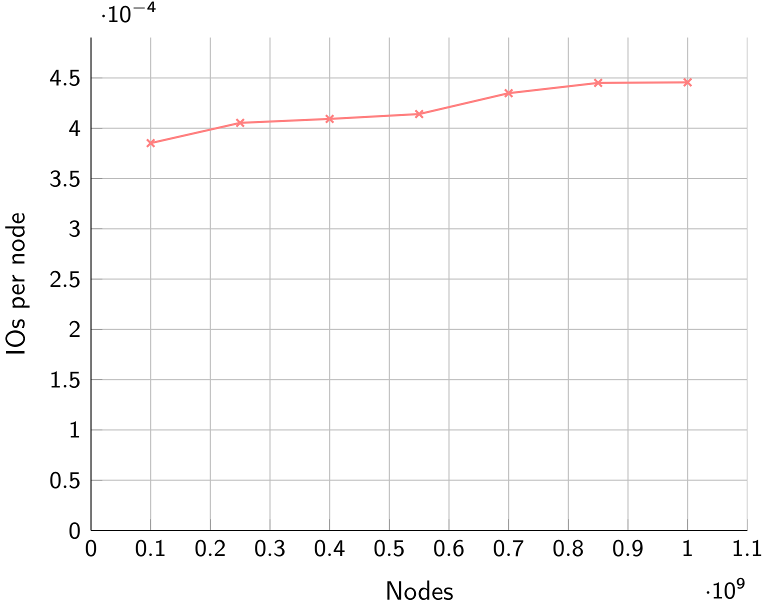
Bisimulation partitioning

- General purpose algorithm
- Topological ordered directed acyclic graphics
- Worst case complexity $O(\mathrm{Sort}(|N| + |E|))$
- Implementation in C++ on top of STXXL
  - Expected IO complexity of $O(\mathrm{Sort}(|N| + |E|))$

Runtime performance



IO Complexity



Application: XML index construction

- 1-Index in $O(\mathrm{Sort}(|N|))$
- A(k)-Index in $O(\mathrm{Sort}(k|N|))$
- F&B-Index by using general algorithm

Future Work

- Cyclic graph partitioning
- Incremental partition maintenance
- Applications in query processing

Conclusion

- Bisimulation partitioning
- External memory
- Applications for XML indexing